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$$x = \frac{\begin{vmatrix} .24561, & .35381, & .36518 \\ .62433, & .90274, & .91143 \\ .17145, & .23642, & .24375 \end{vmatrix}}{\begin{vmatrix} .34622, & .35381, & .36518 \\ .89318, & .90274, & .91143 \\ .22431, & .23642, & .24375 \end{vmatrix}}.$$

If in the numerator determinant two thirds of the third column is subtracted from the first column, and if the second column is subtracted from the third, the value of the determinant is not altered. In the denominator let the middle column be subtracted from each of the others, and then the third column added to the first. The resulting expression for x is

$$x = \frac{\begin{vmatrix} .00216, & .35381, & .01137 \\ .01671, & .90274, & .00869 \\ .00895, & .23642, & .00733 \end{vmatrix}}{\begin{vmatrix} +.00378, & .35381, & .01137 \\ -.00087, & .90274, & .00869 \\ -.00478, & .23642, & .00733 \end{vmatrix}}.$$

The computation can now be made with four-place tables containing all the numbers required on two pages. In computing x it is found in two or three minutes that to three figures its value is $-$ 1.03, which is as far as it is determinate.

THE UNIVERSITY OF CHICAGO,
May 19, 1913.

THE ACCURACY OF INTERPOLATION IN A FIVE-PLACE TABLE OF LOGARITHMS OF SINES.

By A. M. KENYON and G. JAMES, Purdue University.

In a table which gives the logarithms of sines for each minute of angle, the logarithms of sines of angles to tenths of minutes are obtained by interpolation.

The name *Ordinary Interpolation* will be used to denote the process based on the assumption:

(1) As the angle increases from m' to $(m+1)'$ the increment of the logarithm of its sine varies as the increment of the angle; that is,

$$\log \sin (m + n/10)' = \log \sin m' + (n/10)(\log \sin(m+1)' - \log \sin m') \qquad (n = 1, 2, 3, \cdots 9).$$

The error introduced by ordinary interpolation arises from two sources:

(*a*) The *essential* error, due to the assumption (1).

(*b*) The *tabular* error, due to the fact that the tabulated values of $\log \sin m'$ and of $\log \sin(m+1)'$ are in error.

The essential error increases as the angle decreases and makes ordinary interpolation unreliable for small angles. The tabular error is irregular but nowhere in the table does it exceed, in absolute value, one unit in the fifth place.

To make this clear we need to note that whenever an irrational number is approximated by a rational number correct to n decimal places, an error is introduced which in absolute value is less than $5/10^{n+1}$. Thus if we write $\sin 45° = .70711$, the result is too large by an error between .000003 and .000004.

Let $\log \sin m' = a$, $\log \sin(m + 1)' = b$ and let $a - e_1$, $b - e_2$, be the five place values of these same logarithms. Then by ordinary interpolation (assumed to be a correct process since we seek here only the tabular error), the interpolated value of $\log \sin(m + n/10)'$ would be

$$c = a + (n/10)(b - a) = c_1 - e_3$$

where c_1 represents the five place value of c.

But by ordinary interpolation from a five place table, the result will be

$$\begin{aligned} c_2 &= a - e_1 + (n/10)(b - e_2 - a + e_1) \\ &= a + (n/10)(b - a) - [e_1 + (n/10)(e_2 - e_1)] \\ &= c_1 - [e_3 + e_1 + (n/10)(e_2 - e_1)]. \end{aligned}$$

Whence the five place tabular error is

$$c_1 - c_2 = (\bar{n}/10)e_1 + (n/10)e_2 + e_3, \text{ where } n + \bar{n} = 10,$$

and obviously the absolute value of this error is less than .00001.

Actual trial of a very large number of cases (over 1,200) shows that ordinary interpolation below 2° gives the result correct to five places in less than 30 per cent. of all cases; that it gives an error of one in the fifth place in about 50 per cent., and an error of two or more in the fifth place in about 20 per cent. of cases. There is no case above 1° 30′ where the total error is greater than one unit in the fifth place. Above 2° it gives the result correct to five places in from 70 per cent. to 83 per cent. of all cases and in no case tried is the error greater than one unit in the fifth place.

The name *Logarithmic Interpolation* will be used to denote the process based on the following assumption:

(2) For sufficiently small angles, the sine varies as the angle; which gives, if m and n denote measures of angles in the same unit,

$$\log \sin(m + n/10) = \log \sin m + \log(m + n/10) - \log m.$$

Here the essential error decreases with the angle; the tabular error as before is irregular but nowhere in the table exceeds, in absolute value, one unit in the fifth place.

Actual trial of all cases (to tenths of minutes) shows that in no case below 5° 45′ does logarithmic interpolation introduce a total error greater than one in the

fifth place.[1] Below 5° it gives the result correct to five places in about 60 per cent. of all cases and shows an error of one in the fifth place in about 40 per cent. of cases.

Below 2°, logarithmic interpolation is certainly preferable to ordinary interpolation for three reasons: (1) it gives the correct result in a larger number of cases; (2) it gives an error in the fifth place in a smaller number of cases; (3) it never gives an error greater than one in the fifth place.

Above 2°, ordinary interpolation is preferable for the same three reasons. All cases between 1° 30′ and 2° 15′ were examined.

An examination of 200 cases in the neighborhood of 84° shows that ordinary interpolation will give the result correct to five places in about 83 per cent., and will show an error of one in the fifth place in 17 per cent. of cases.

Below is a table of results of the statistical investigation.

Interval of Angle.	Number of Cases Examined.	Per Cent. of Cases Giving Result Correct to 5 Places.		Per Cent. of Cases Showing an Error of One in the 5th Place.		Per Cent. of Cases Showing an Error of Two or More in the Fifth Place.	
		Ordinary.	Logarithmic.	Ordinary.	Logarithmic.	Ordinary.	Logarithmic.[2]
1′–1°	45	Fails	60	Large	40		0
1° –1° 30′	97	33		57		10	0
1° 30′–1° 45′	All	55	63	45	37	0	0
1° 45′–2°	All	63	69	37	31	0	0
2° –2° 15′	All	71	62	29	38	0	0
2° 15′–3°	55	78	60	22	40	0	0
3° –4°	55	80		20		0	0
4° –5°						0	0
5° –5° 45′	All		66		34	0	0
5° 45′–6°	All		64		36		[3]

A THEOREM ABOUT ISOGONAL CONJUGATES.

By DAVID F. BARROW, Harvard University.

As an introduction let us recall two well-known theorems of elementary plane geometry:

THEOREM I. *Given a triangle $A_1A_2A_3$ (Fig. 1) and any point P not a vertex, join P to the three vertices and reflect each of the three lines thus drawn in the bisector of the angle at the corresponding vertex. The three reflected lines will meet in a point P', called the isogonal conjugate of P.*

THEOREM II. *Given a triangle and a point marked at random on each side. If three circles be drawn, one through each vertex and the two adjacent marked points, these three circles meet in a common point.*

[1] The first error of two in the fifth place enters at 5° 53′.7

[2] All cases examined for this column.

[3] At 5° 53′.7 the first error of 2 in the fifth place enters.